

CLASA a IV-a

Here are some suggestions to help you do your best:

- Read carefully each question and think about the answer before choosing your response.

RULES

- Part I has four multiple choice exercises.
- Part II must be solved into English
- Part III must be translated into English, and then solved in English as well.

PART I.

1. In the figure, we see an island with a funny shape and several frogs.



How many of these frogs are sitting on the island?

- A 5
- B 6
- C 7
- D 8
- E 9

2. A number has two digits. The product of the digits of this number is 15. What is the sum of the digits of this number?

- A 2
- B 4
- C 6
- D 7
- E 8

3. Tom added all even numbers from 2 to 100. Alice added all odd numbers from 1 to 99. Then Joe subtracted Alice's result from Tom's result. What is Joe's result?

- A 50
- B 150
- C 10
- D 200
- E 100

4. Lisa has 90 marbles. Anna has 10 more marbles than Lisa but 50 less marbles than Olle. How many marbles do they have together?

- A 100
- B 150
- C 240
- D 250
- E 340

PART II.

In a speed skating competition, 10 racers reached the final. Tom beat three racers more than beat him. Which place did Tom end up in?

PART III.

Într-o livadă sunt 128 de pomi fructiferi: meri, pruni și caiși. Merii sunt cu 10 mai puțini decât dublul prunilor, iar numărul total de meri și pruni este cu 8 mai mare decât triplul numărului de caiși. Câți pomi fructiferi din fiecare fel sunt în această livadă.

CLASA a V-a

Here are some suggestions to help you do your best:

- Read carefully each question and think about the answer before choosing your response.

RULES

- Part I has four multiple choice exercises.
- Part II must be solved into English
- Part III must be translated into English, and then solved in English as well.

PART I.

1. The figure shows an addition where the numbers are coded by letters. Equal letters represent equal digits, and different letters represent different digits. Which digit does the letter X represent?

$$\begin{array}{r} X \\ X \\ \hline YY \\ \hline ZZZ \end{array}$$

- A 2
- B 3
- C 4
- D 5
- E 6

2. Two cats, Tom and Bob, caught together 42 mice in three days. Each day, Tom caught twice as many mice as in the previous day, while Bob caught two more mice than in the previous day. However, both cats caught the same number of mice for the three days. How many mice did Tom and Bob catch together during the first day?

- A 5
- B 7
- C 8
- D 9
- E 10

3. Which of the following is neither a square number nor a cube number

- A 6^{13}
- B 5^{12}
- C 4^{11}
- D 3^{10}
- E 2^9

4. How many solutions does the equation $2^{2x} = 4^{x+1}$ have?

- A 0
- B Infinitely many
- C 2
- D 1
- E 3

PART II.

A father tells his son: "In four years, I will be five times older than you." His mother adds: "Yes, and in 22 years, Dad will be only twice as old as you". How old are the father and the son now?

PART III.

Aflați patru numere știind că suma lor este egală cu 55, primul număr este jumătate din cel de al doilea, al treilea număr este media aritmetică a primelor două, iar al patrulea număr este dublul diferenței dintre al doilea și al treilea.

CLASA a VI-a

Here are some suggestions to help you do your best:

- Read carefully each question and think about the answer before choosing your response.

RULES

- Part I has four multiple choice exercises.
- Part II must be solved into English
- Part III must be translated into English, and then solved in English as well.

PART I.

1. Which of the following numbers is closest to $2,015 \times 510,2$?

- A 0,1
- B 1
- C 10
- D 100
- E 1000

2. A triangle has sides of lengths 6, 10 and 11. An equilateral triangle has the same perimeter. What is the side length of the equilateral triangle?

- A 18
- B 11
- C 10
- D 9
- E 6

3. Five points lie on a line. Alex finds the distances between every possible pair of points. He obtains, in increasing order, 2, 5, 6, 8, 9, k, 15, 17, 20 and 22. What is the value of k?

- A 10
- B 11
- C 12
- D 13
- E 14

4. On a map a 12 cm length represents 72 km. How many km does a 17 cm length represent?

- A 6
- B 102
- C 104
- D 12
- E 864

PART II.

A box contains gold coins. If the coins are equally divided among six people, four coins are left over. If the coins are equally divided among five people, three coins are left over. If the box holds smallest number of coins that meets these two conditions, how many coins are left when equally divided among seven people?

PART III.

Fie punctele $A_0, A_1, A_2, \dots, A_n$ situate în această ordine pe aceeași dreaptă d astfel încât $A_0A_1 = 1$ cm, $A_1A_2 = 2$ cm, $A_2A_3 = 2^2$ cm, ..., $A_{n-1}A_n = 2^{n-1}$ cm.

- 1) Determinați numărul natural p astfel încât $A_0A_p = 2047$ cm.
- 2) Dacă M este mijlocul segmentului $[A_2A_{12}]$ și N este mijlocul segmentului $[A_4A_{10}]$ determinați lungimea segmentului $[MN]$.

CLASA a VII-a

Here are some suggestions to help you do your best:

- Read carefully each question and think about the answer before choosing your response.

RULES

- Part I has four multiple choice exercises.
- Part II must be solved into English
- Part III must be translated into English, and then solved in English as well.

PART I.

1. In the triangle ABC of area 120 cm^2 , the ratio between the length of one side and the corresponding height is 3:5. What is this height, in cm?

- A 20
- B 24
- C 12
- D 16
- E 4

2. In the trapezoid PQRS, the sides PQ and SR are parallel. Angle RSP is 120° and $RS = SP = 1 \text{ PQ}$. What is the size of the angle PQR?

- A 15°
- B $22,5^\circ$
- C 25°
- D 30°
- E 45°

3. There are 30 problems in the Mathematics competition for pupils. Two points are scored for each correct answer. If a pupil omits to solve a problem, he/she gets 0 points on it. One point is lost by a pupil for each wrong answer. Mark answered 25 problems and got 5 points. How many wrong answers did he have?

- A 25
- B 20
- C 15
- D 10
- E 5

4. The integers x and y with $x > y > 0$ satisfy $x + y + xy = 80$. What is x ?

- A 8
- B 10
- C 15
- D 18
- E 26

PART II.

Consider the set of all fractions $\frac{x}{y}$, where x and y are relatively prime integers.

How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%.

PART III.

În triunghiul dreptunghic ABC cu $m(\angle A) = 90^\circ$ și $m(\angle B) = 30^\circ$ se construiește bisectoarea unghiului C , care intersectează latura AB în punctul D . Prin D se construiește $DE \perp BC$, $E \in BC$ și prin E se duce $EF \parallel CD$, $F \in AB$. Știind că $AB = 36$ cm, calculați lungimea segmentului EF .

CLASA a VIII-a

Here are some suggestions to help you do your best:

- Read carefully each question and think about the answer before choosing your response.

RULES

- Part I has four multiple choice exercises.
- Part II must be solved into English
- Part III must be translated into English, and then solved in English as well.

PART I.

1. If $y+4 = (x-2)^2$, $x+4 = (y-2)^2$, and $x \neq y$, what is the value of x^2+y^2 ?

- A 10
- B 15
- C 20
- D 25
- E 30

2. What is the volume of a cube whose surface area is twice that of a cube with volume 1?

- A $\sqrt{2}$
- B 2
- C $2\sqrt{2}$
- D 4
- E 8

3. Pick two consecutive positive integer numbers whose sum is less than 100. Square both of those integers and find the difference of the squares. Which number could be the difference?

- A 2
- B 64
- C 79
- D 96
- E 131

4. An aquarium has a rectangular base that measures 100 cm by 40 cm and has a height of 50 cm. The aquarium is filled with water to a depth of 37 cm. A rock with volume 1000 cm^3 is then placed in the aquarium and completely submerged. By how many cm does the water level rise?

- A 0,25
- B 0,5
- C 1
- D 1,5
- E 2,5

PART II.

The sum of the first m positive odd integers is 212 more than the sum of the first n positive even integers. What is the sum of all possible values of n ?

PART III.

Fie $VABC$ o piramidă triunghiulară regulată și M un punct variabil în planul bazei. Perpendiculara în M pe planul bazei intersectează planele fețelor laterale VBC , VAC , respectiv VAB în punctele A_1 , B_1 , respectiv C_1 . Arătați că suma $MA_1 + MB_1 + MC_1$ este constantă.

PUNCTAJ:

| Sub.I | | | | Sub.II | total | Sub.II | total | Sub.III | total | Sub.III | total |
|-------|------|------|------|--------|-------|--------|-------|---------|-------|---------|-------|
| 1 | 2 | 3 | 4 | Mate | mate | Engl | engl | Mate | mate | Engl | engl |
| 0,50 | 0,50 | 0,50 | 0,50 | 2p | 4p | 1p | 1p | 2p | 2p | 2p | 2p |

CLASA a IV-a

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| B | E | A | E |

CLASA a V-a

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| E | C | A | A |

CLASA a VI-a

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| E | D | E | B |

CLASA a VII-a

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| A | D | C | E |

CLASA a VIII-a

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| B | C | C | A |

Mark scheme - Part III

Year 4:

In an orchard there are 128 fruit trees: apple trees, plum trees and apricot trees. There are ten less apple trees than the double of plum trees, and the total number of apple and plum trees is eight more than the triple of the number of apricot trees. How many fruit trees of each kind are there in this orchard?

Year 5:

Find four numbers knowing that their sum is equal to 55. The first number is half of the second one, the third number is the arithmetic mean of the first two numbers, and the fourth number is double the difference between the second and the third.

Year 6:

Let $A_0, A_1, A_2, \dots, A_n$ be the points situated in this order on the same line d so that $A_0A_1 = 1$ cm, $A_1A_2 = 2$ cm, $A_2A_3 = 2^2$ cm, ..., $A_{n-1}A_n = 2^{n-1}$ cm.

- 1) Find natural number p so that $A_0A_p = 2047$ cm.
- 2) If M is the midpoint of segment $[A_2A_{12}]$ and N is the midpoint of segment $[A_4A_{10}]$ find the length of segment $[MN]$.

Year 7:

In the right triangle ABC , where $m(\sphericalangle A) = 90^\circ$ and $m(\sphericalangle B) = 30^\circ$, draw the bisector of angle C , which intersects side AB in point D . Through point D draw $DE \perp BC$, $E \in BC$ and through point E draw $EF \parallel CD$, $F \in AB$. Knowing that $AB = 36$ cm, find the length of segment EF .

Year 8:

Let $VABC$ be a regular triangular pyramid and M a variable point on its base. The perpendicular in M on the base intersects the lateral sides' planes VBC , VAC , and VAB in points A_1 , B_1 , and C_1 . Prove that the sum $MA_1 + MB_1 + MC_1$ is constant.

BAREM

clasa a IV-a

III (20p)

Fie a, b, c numărul de mere, pruni, cași,

$$a = 2b - 10$$

$$a + b = 8 + 3c$$

$$a + b + c = 128$$

----- 5p

$$\Rightarrow 8 + 3c + c = 128$$

$$8 + 4c = 128$$

$$c = 30 \text{ cași}$$

----- 5p

$$a + b = 98$$

$$2b + b - 10 = 98$$

$$3b = 108$$

$$b = 36 \text{ pruni}$$

----- 7p

$$a = 62 \text{ mere}$$

----- 3p

Orice soluție corectă se punctează cu punctajul maxim : 20p

BAREM

Clasa a \bar{V} -a
III (20p)

Fie a, b, c, d cele 4 numere

$$a+b+c+d = 55 \quad \text{---} \quad 2p$$

$$a = \frac{b}{2} \Rightarrow b = 2a \quad \text{---} \quad 2p$$

$$c = \frac{a+b}{2} \Rightarrow c = \frac{3a}{2} \quad \text{---} \quad 4p$$

$$d = 2(b-c) \Rightarrow d = 2\left(2a - \frac{3a}{2}\right) = a \quad \text{---} \quad 4p$$

$$a + 2a + \frac{3a}{2} + a = 55$$

$$\frac{11a}{2} = 55 \quad \text{---} \quad 6p$$

$$a = 10 \quad \text{---} \quad 2p$$

$$b = 20$$

$$c = 15$$

$$d = 10$$

Orice soluție corectă se punctează cu punctajul maxim: 20p.

BAREM

III Clasa a VI-a
(20p)

$$1) A_0 A_p = A_0 A_1 + A_1 A_2 + \dots + A_{p-1} A_p$$

$$= 1 + 2 + 2^2 + \dots + 2^{p-1} = 2^p - 1 \quad - \frac{5p}{-}$$

$$\Rightarrow 2^p - 1 = 2047 \quad \Rightarrow 2^p = 2048 \quad \Rightarrow p = 11 \quad - \frac{5p}{-}$$

$$2) A_0 A_m = 2^m - 1, \quad m \in \mathbb{N}$$

$$A_2 A_{12} = A_0 A_{12} - A_0 A_2 = 2^{12} - 2^2 \text{ cm} \quad - \frac{2p}{-}$$

$$M \text{ mijloc } [A_2 A_{12}] \Rightarrow A_2 M = M A_{12} = 2^{11} - 2 = 2046 \text{ cm} \quad - \frac{2p}{-}$$

$$A_4 A_{10} = A_0 A_{10} - A_0 A_4 = 2^{10} - 2^4 \text{ cm} \quad - \frac{2p}{-}$$

$$N \text{ mijloc } [A_4 A_{10}] \Rightarrow A_4 N = N A_{10} = 2^9 - 2^3 = 504 \text{ cm} \quad - \frac{2p}{-}$$

$$MN = A_0 M - A_0 N = (A_0 A_2 + A_2 M) - (A_0 A_4 + A_4 N)$$

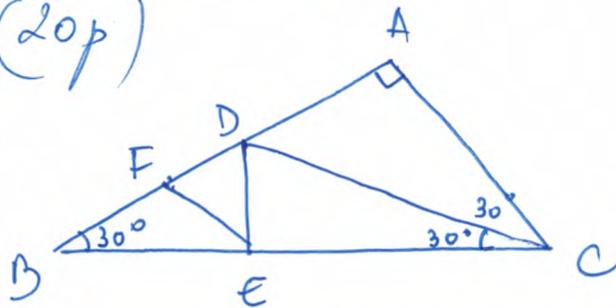
$$= (2^2 - 1 + 2046) - (2^4 - 1 + 504) = 1530 \text{ cm} \quad \frac{2p}{-}$$

Orice soluție corectă se punctează cu punctajul maxim: 20p

BAREM

Clasa a VII-a

III (20p)



$$\Delta ABC, m(\hat{A}) = 90^\circ, m(\hat{B}) = 30^\circ \xrightarrow{T_{30^\circ}} BC = 2AC \quad \text{--- 4p}$$

$$(CD \text{ bis } \xrightarrow{T_{\text{bis}}} \frac{AD}{BD} = \frac{AC}{BC} = \frac{1}{2} \quad \text{--- 4p}$$

$$\Rightarrow \left. \begin{aligned} \frac{AD+BD}{BD} &= \frac{3}{2} \\ AD+BD &= 36 \end{aligned} \right\} \Rightarrow BD = 24 \text{ cm} \quad \text{--- 4p}$$

$$(CD \text{ bis } \Rightarrow \Delta BDC \text{ tris } \Rightarrow CD = BD = 24 \text{ cm} \quad \text{--- 4p}$$

$$\left. \begin{aligned} DE \perp BC &\Rightarrow E \text{ mijl } [BC] \\ EF \parallel CD \end{aligned} \right\} \Rightarrow EF \text{ lm in } \Delta BD$$

$$\Rightarrow EF = \frac{CD}{2} = 12 \text{ cm} \quad \text{--- 4p}$$

Orice soluție corectă se punctează cu punctajul maxim: 20p

BAREM

Clasa a VIII-a

III (20p)

$$\left. \begin{array}{l} \text{Fie } ME \perp BC \\ MA_1 \perp (ABC) \end{array} \right\} \stackrel{T_3}{\Rightarrow} A_1E \perp BC \quad \underline{\underline{2p}}$$

$$\left. \begin{array}{l} (VBC) \cap (ABC) = BC \\ ME \perp BC, ME \in (ABC) \\ A_1E \perp BC, A_1E \in (VBC) \end{array} \right\} \Rightarrow m(\widehat{(VBC), (ABC)}) = m(\widehat{A_1EM}) = \alpha^\circ \quad \underline{\underline{4p}}$$

$\alpha^\circ =$ Măsură unghiului format de o față laterală cu planul bazei $\underline{\underline{2p}}$

$$\Rightarrow MA_1 = ME \cdot \operatorname{tg} \alpha \quad \underline{\underline{4p}}$$

Analog, $MF \perp AC, MB_1 = MF \cdot \operatorname{tg} \alpha$
 $MG \perp AB, MC_1 = MG \cdot \operatorname{tg} \alpha$ $\underline{\underline{4p}}$

$$\Rightarrow MA_1 + MB_1 + MC_1 = (ME + MF + MG) \cdot \operatorname{tg} \alpha = h_{\Delta ABC} \cdot \operatorname{tg} \alpha = \text{const} \quad \underline{\underline{4p}}$$

Orice soluție corectă se punctează cu punctajul maxim: 20p

BAREM
Clasa a IV-aII (20p)

Fie a numărul de concurenți care îl înving pe Tom. - 5p

$a+3$ = numărul de concurenți care sunt învinsi de Tom. - 5p

$$a+1+a+3 = 10$$

$$\Rightarrow a = 3$$

Tom a ieșit pe locul ~~4~~ patru. - 2p

BAREM

clasa a v-a

II
(20p)Fie t - vârsta tatălui f - vârsta fiului

$$t+4=5(f+4) \Rightarrow t=5f+16 \quad \text{---} \quad 5p$$

$$t+22=2(f+22) \Rightarrow t=2f+22 \quad \text{---} \quad 5p$$

$$5f+16=2f+22 \quad \text{---} \quad 5p$$

$$3f=6$$

$$f=2$$

$$t=26$$

$$\text{---} \quad 5p$$

Orice soluție corectă se punctează
cu punctajul maxim: 20p.

BAREM

clasa a VI-a

II (20p)

Fie m numărul de monede

$$m = 6c_1 + 4$$

----- 3p.

$$m = 5c_2 + 3$$

----- 3p.

$$\Rightarrow m+2 = 6(c_1+1) \Rightarrow m+2 \div 6$$

----- 3p.

$$m+2 = 5(c_2+1) \Rightarrow m+2 \div 5$$

----- 3p.

$$\Rightarrow m+2 \div 30$$

----- 3p.

$$\Rightarrow m+2 = 30$$

----- 3p.

$$m = 28, 28 \div 4 \Rightarrow k = 0$$

----- 2p.

BAREM

clasa a VII-a

II (20p)

$$\frac{x+1}{y+1} = \frac{11x}{10y}$$

5p

$$\Rightarrow 10y(x+1) = 11x(y+1)$$

$$\Rightarrow xy + 11x - 10y = 0$$

$$(y+11)(10-x) = 110$$

5p

$$y+11 \in \{22, 55, 110\} \Rightarrow y \in \{11, 44, 99\}$$

$$10-x \in \{5, 2, 1\} \Rightarrow x \in \{5, 8, 9\}$$

5p

$\frac{x}{y}$ - fracție ireductibilă

$$\Rightarrow \frac{x}{y} = \frac{5}{11}$$

O singură fracție îndeplinește condițiile din enunț.

5p

BAREM

clasa a VIII-a

II (20p)

$$(1+3+5+\dots+2m-1) - (2+4+\dots+2m) = 212 \quad \underline{\underline{2p}}$$

$$m^2 - m(m+1) = 212 \quad | \cdot 4$$

$$4m^2 - (4m^2 + 4m + 1) = 847$$

$$(2m - 2m - 1)(2m + 2m + 1) = 847 \quad \underline{\underline{4p}}$$

$$2m - 2m - 1 < 2m + 2m + 1 \quad \text{- nr impare}$$

$$847 = 7 \cdot 11^2$$

$$\text{I} \quad 2m - 2n - 1 = 1$$

$$2m + 2n + 1 = 847$$

$$\Rightarrow 4m + 2 = 846 \Rightarrow m = 211 \quad \underline{\underline{4p}}$$

$$\text{II} \quad 2m - 2n - 1 = 7$$

$$2m + 2n + 1 = 121$$

$$\Rightarrow 4m + 2 = 114 \Rightarrow m = 28 \quad \underline{\underline{4p}}$$

$$\text{III} \quad 2m - 2n - 1 = 11$$

$$2m + 2n + 1 = 77$$

$$\Rightarrow 4m + 2 = 66 \Rightarrow m = 8 \quad \underline{\underline{4p}}$$

$$S = 211 + 28 + 8 = 247 \quad \underline{\underline{2p}}$$